Analytical diagnostics for interpreting sodium lines in exoplanetary atmospheres

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Heng, Wyttenbach, Lavie, Sing, Ehrenreich & Lovis (2015, ApJ Letters, 803, L9)

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EEG EXOPLANETS & EXOCLIMES GROUP

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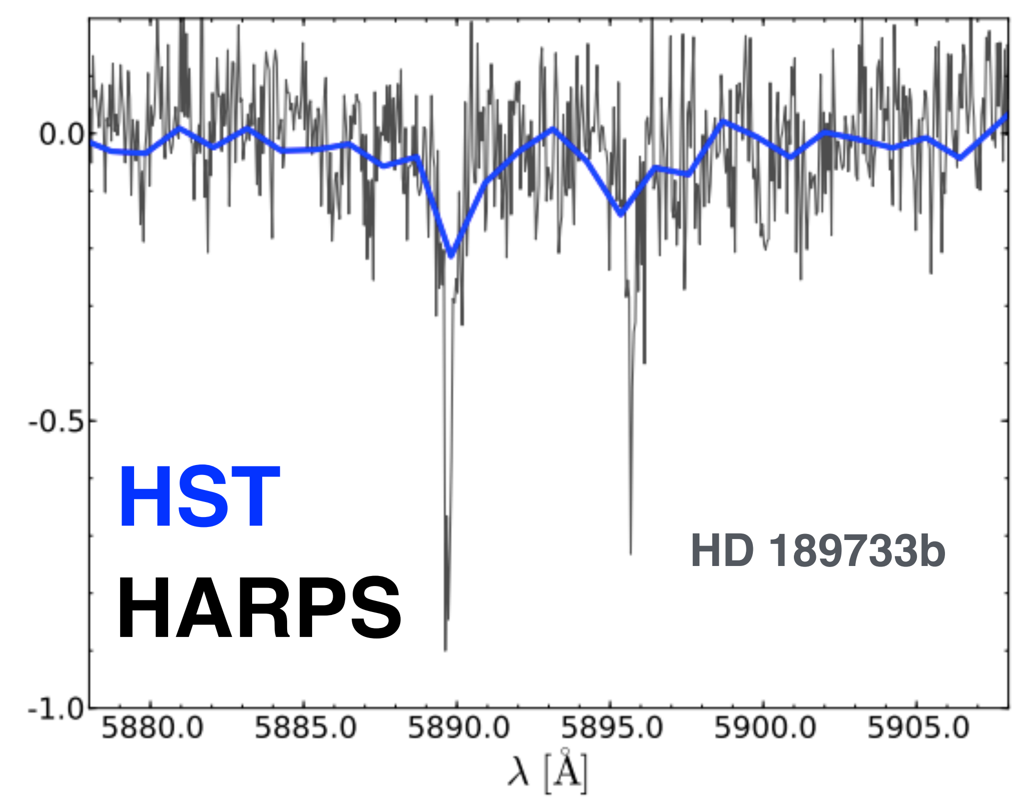
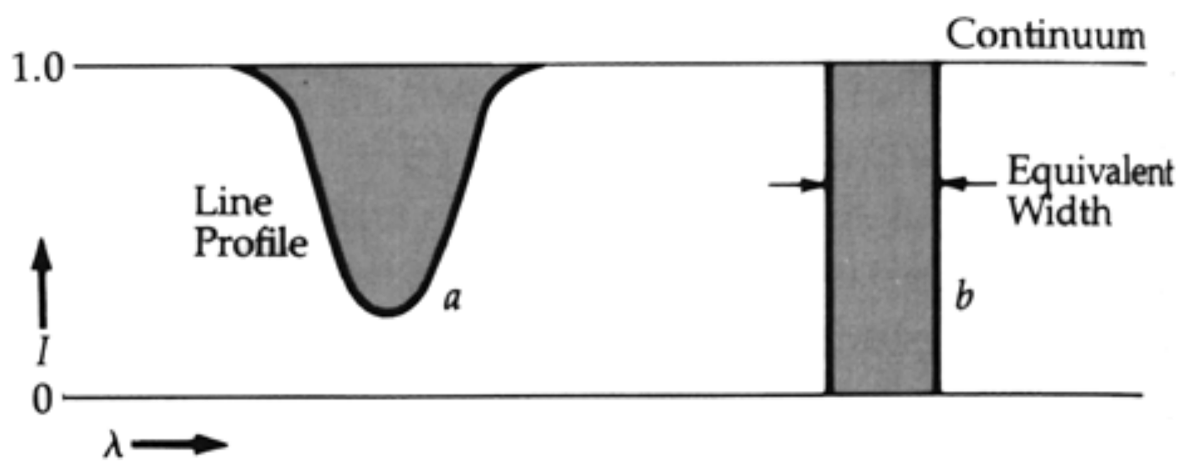
Why is the sodium doublet interesting?

- Very large cross section (don't need a lot of sodium atoms).
- Resides in visible wavelength range (about 0.6 microns).
- Used to make first detection of an exoplanetary atmosphere.
  (Charbonneau et al. 2002)
- Ultra-high-resolution spectrographs (R=100,000) are opening up new perspectives on what can be extracted from the data.
  (Wyttenbach et al. 2015)



The equivalent width



$$W_\lambda = \frac{F_c - F}{F_c}$$

$$W = \int W_\lambda \, d\lambda$$

Is a resolution-independent way to compare the strength of a line

Key point: can measure it without knowing the size of the object

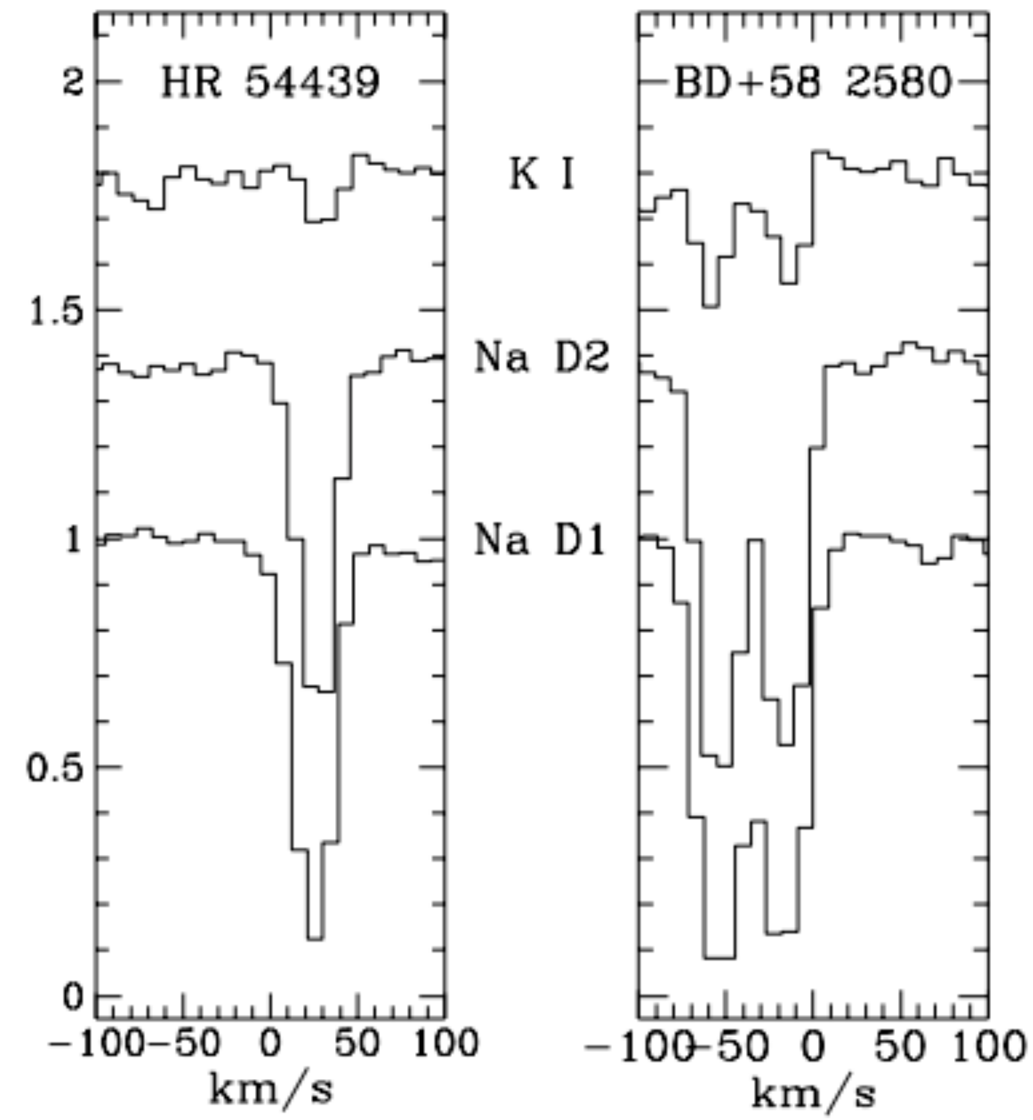
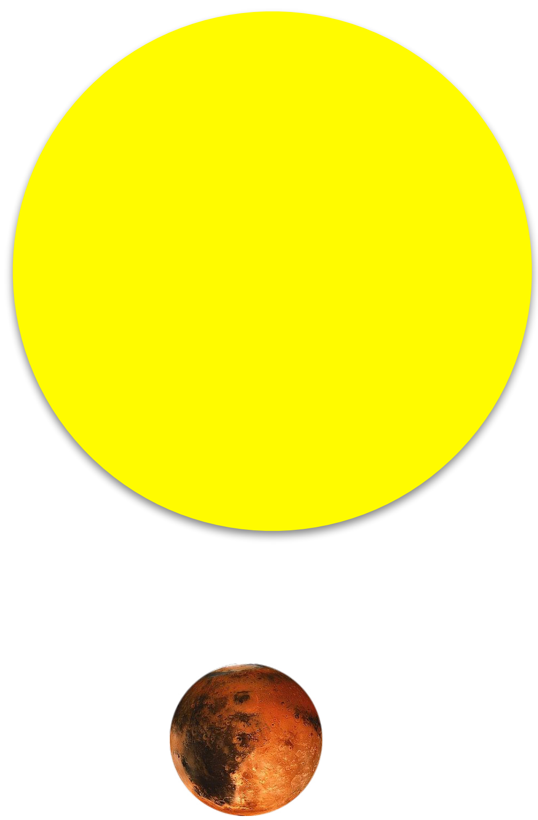


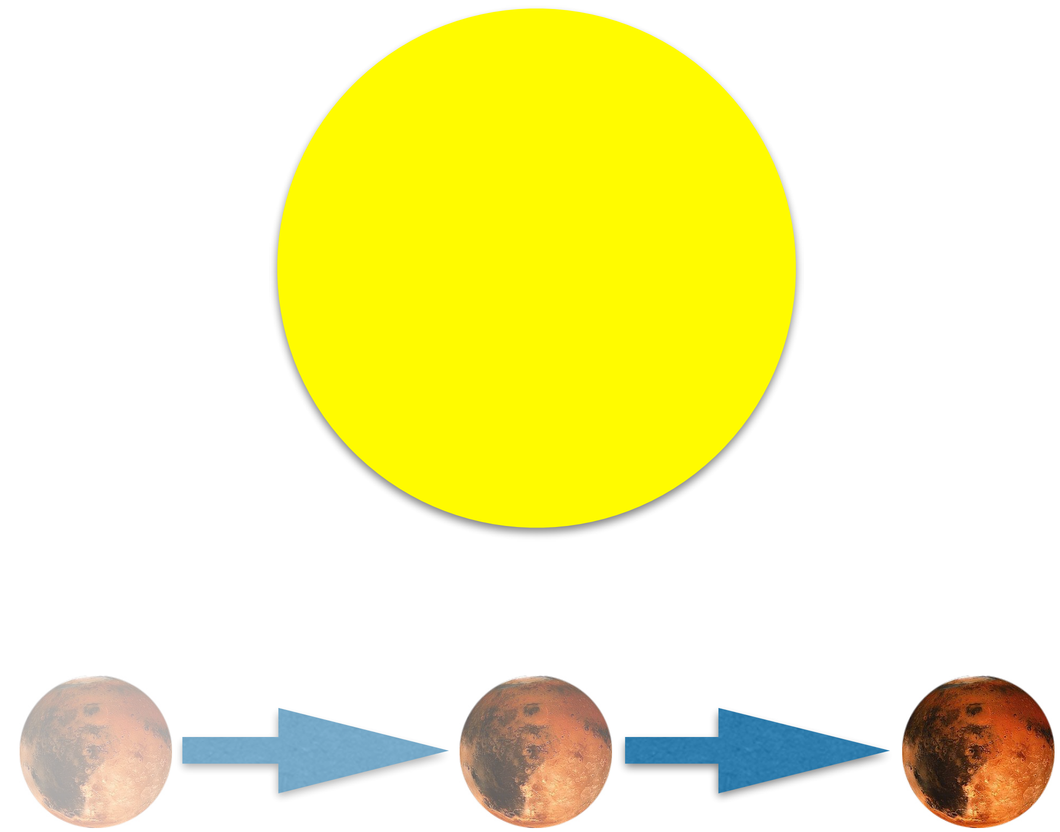
Fig. 3. Examples of the normalized spectra in the region of Na I and K I lines. The spectra have been vertically offset for clarity. Velocity is heliocentric.

ISM sodium and potassium lines

Munari & Zwitter (1997)

Absorption depth vs. transit radius





Record the absorption spectrum of exoplanet at a fixed moment in time

No knowledge of radius!
Only absorption depth
(flux normalised by continuum)

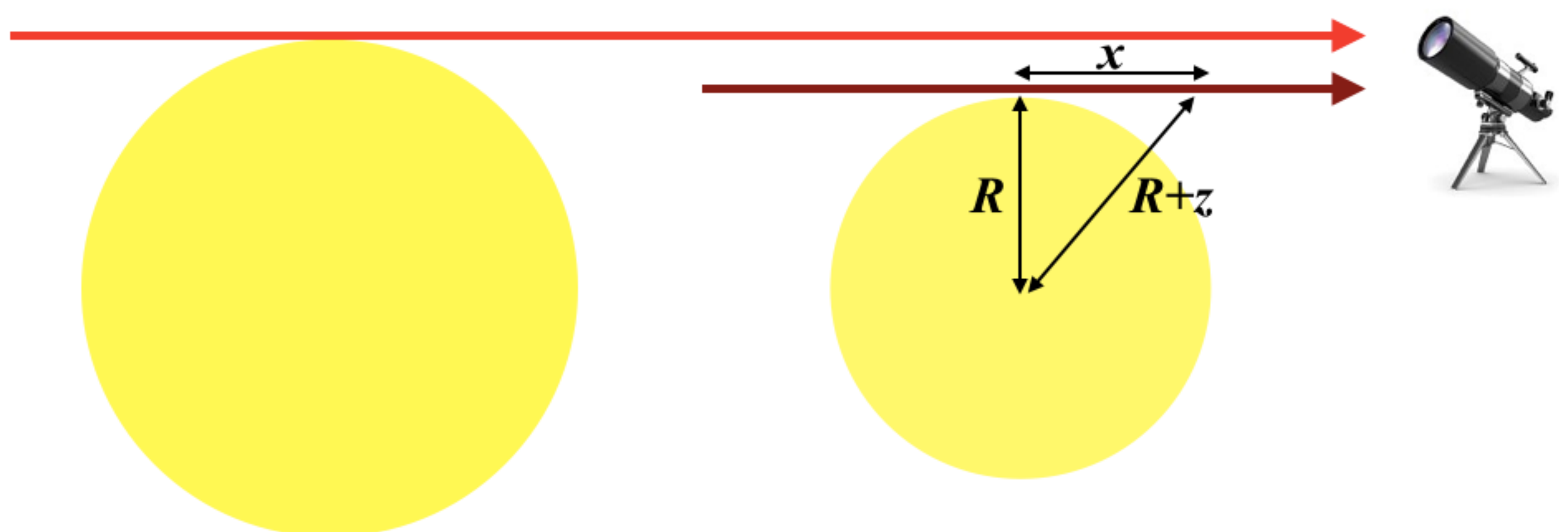
Record change in flux across time at one wavelength

Can derive radius of exoplanet relative to star, but no wavelength information obtained

Conclusion: these are independent observables!

Two classic studies:

Fortney (2005) & Lecavelier des Etangs et al. (2008)



line center
transit radius: R_0
wavelength: λ_0

line wings
transit radius: R
wavelength: λ

$$n = n_{\rm ref} \exp\left(-\frac{z}{H}\right)$$

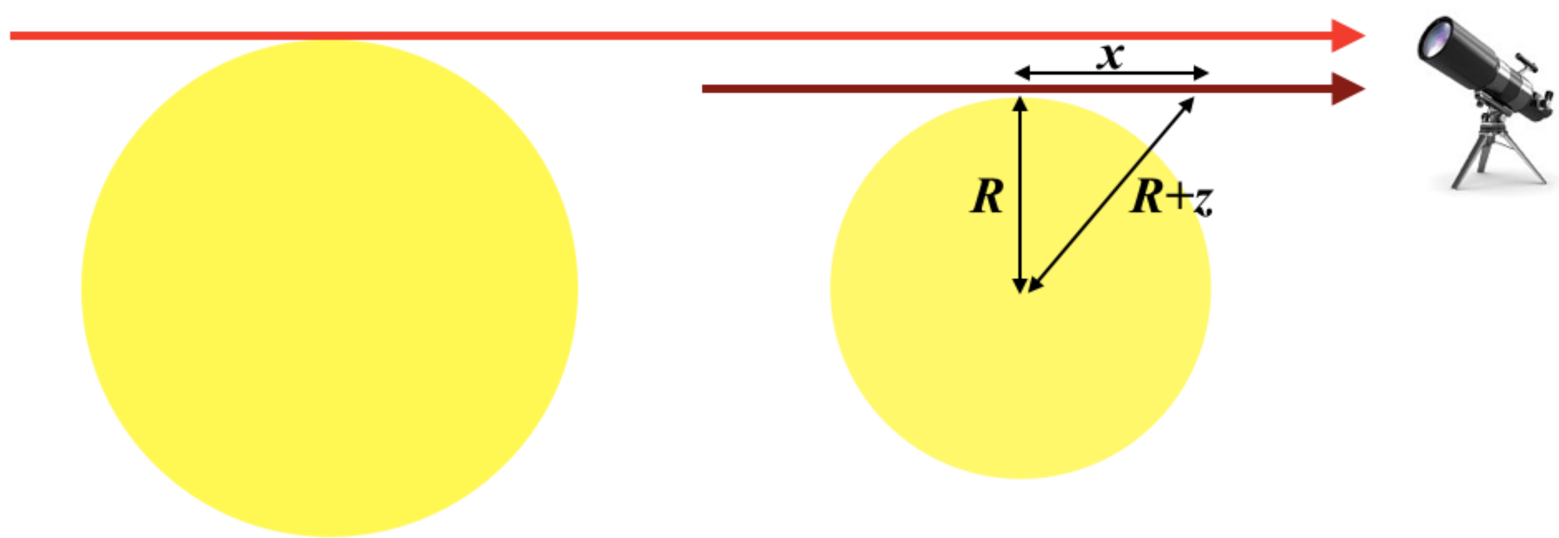
assume hydrostatic equilibrium + isothermality

$$\tau = n_{\rm ref}\,\sigma \int_{-\infty}^{+\infty} \exp\left(-\frac{x^2}{2HR}\right) dx$$
$$= n_{\rm ref}\,\sigma\,(2\pi HR)^{1/2},$$

integrate along transit chord to get optical depth

Two classic studies:

Fortney (2005) & Lecavelier des Etangs et al. (2008)



line center
transit radius: R_0
wavelength: λ_0

line wings
transit radius: R
wavelength: λ

integrate Fortney's expression and get temperature

$$T = \frac{mg}{k_\mathrm{B}} \frac{\partial z}{\partial \lambda} \left[\frac{\partial (\ln \sigma)}{\partial \lambda} + \frac{1}{2} \frac{\partial (\ln R)}{\partial \lambda} \right]^{-1}$$

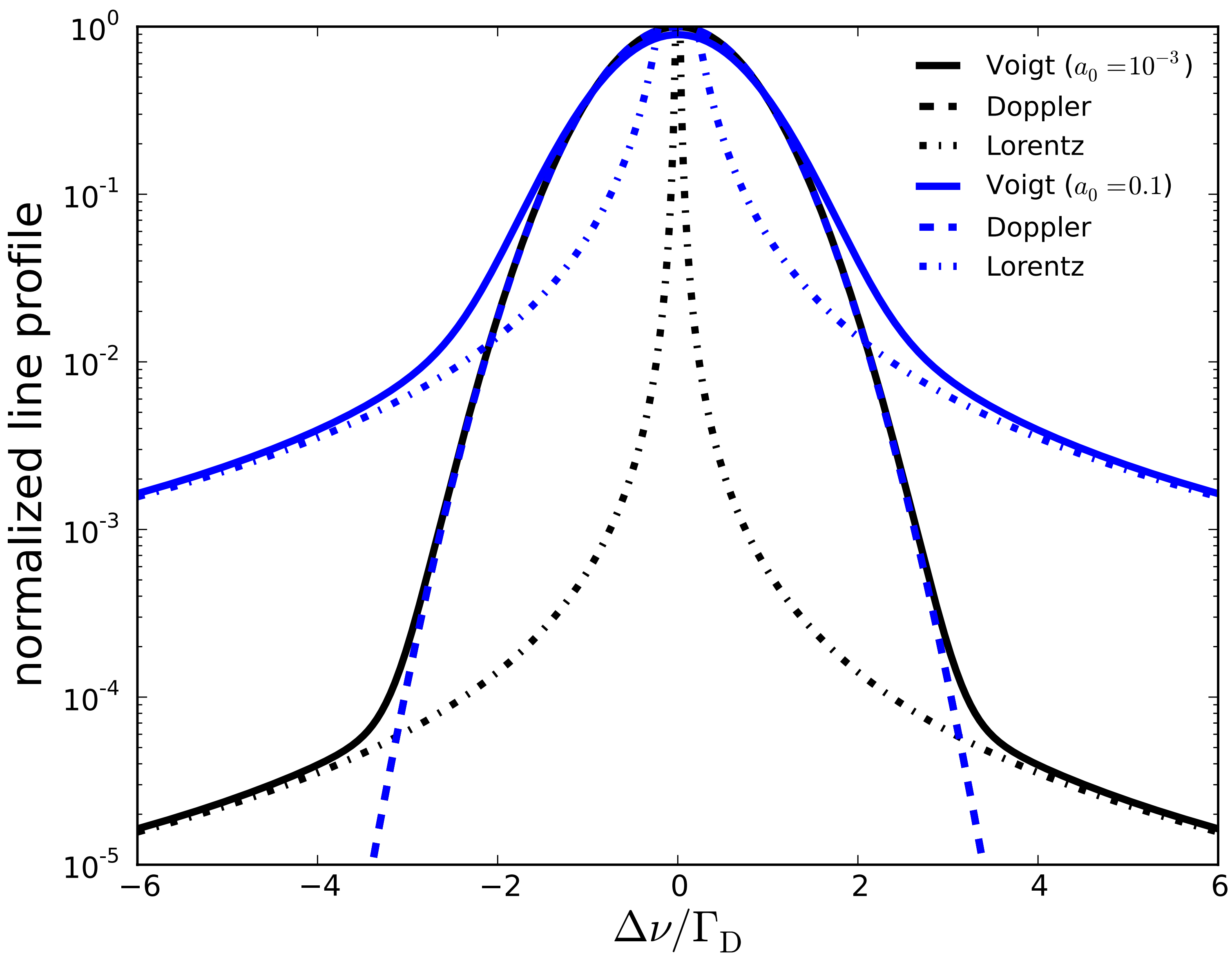
observed — cross section — negligible

Flaw:
may be used to analyse a **continuum** (Rayleigh scattering), but **not for lines**!

What do we wish to improve?

- Need a theory for jointly interpreting absorption depth and transit radius.
- The formula is isothermal. We cannot formally use it to infer a temperature-pressure profile for the atmosphere.
- Would like an approach **for lines** and not the continuum.
- **Easy to use**, analytical formulae.
- Wish to derive **temperatures** and **number densities** at line centre and wing. Wish to know **temperature gradient**.

How do we describe the sodium line profile/shape?



Key point:
damping parameter
of Voigt profile is small

Line center = Doppler
Line wing = Lorentz

damping parameter:
controls line shape

$$a_0 = \frac{\Gamma_L}{2\Gamma_D} \approx 10^{-3}\left(\frac{T}{10^3\,\mathrm{K}}\right)^{-1/2}\left(\frac{m}{2m_H}\right)^{1/2}$$

Lorentz width (Γ_L)

Doppler width (Γ_D)

A common misconception associated with hydrostatic equilibrium

$$\frac{\partial P}{\partial z} = -\rho g \quad \xrightarrow{\text{use ideal gas law}} \quad n = n_{\text{ref}} \exp\left(-\frac{z}{H}\right)$$

hydrostatic equilibrium implies exponential density profile

A common misconception associated with hydrostatic equilibrium

$$\frac{\partial P}{\partial z} = -\rho g \quad \xrightarrow{\text{use ideal gas law}} \quad n = n_{\rm ref} \exp\left(-\frac{z}{H}\right)$$

hydrostatic equilibrium implies exponential density profile

Wrong! Only correct when it is isothermal

$$n = \begin{cases} n_{\rm ref}\left(1 + \dfrac{T'z}{T_{\rm ref}}\right)^{-(b+1)}, & \dfrac{\partial T}{\partial z} > 0, \\ n_{\rm ref}\left(1 - \dfrac{T'z}{T_{\rm ref}}\right)^{b-1}, & \dfrac{\partial T}{\partial z} < 0, \end{cases}$$

T': magnitude of temperature gradient

b: ratio of non-isothermal to isothermal scale height

Correct formulae (constant temperature gradient)

Theory & Application

main diagnostic

$$\frac{T_1}{T_2} = \frac{\Delta R_1}{\Delta R_2}$$

measure transit radii at line wing and center for each sodium line (4 measurements in total)

Novel attributes:

- self-consistent temperature gradient
- number densities (absolute abundances)

diagnostics applied to HD 189733b

$$\frac{\partial T}{\partial z} = 0.4376 \pm 0.0154 \text{ K km}^{-1}$$

$$T_1 = 2460 \pm 86 \text{ K}, \quad T_{0,1} = 4306 \pm 151 \text{ K},$$
$$T_2 = 3336 \pm 117 \text{ K}, \quad T_{0,2} = 5870 \pm 206 \text{ K}.$$
$$n_1 = (1.439 \pm 0.051) \times 10^4 \text{ cm}^{-3},$$
$$n_2 = (1.219 \pm 0.043) \times 10^4 \text{ cm}^{-3},$$
$$n_{0,1} = 3.990 \pm 0.140 \text{ cm}^{-3},$$
$$n_{0,2} = 3.128 \pm 0.110 \text{ cm}^{-3},$$

HARPS data by Wyttenbach et al. (2015)

Temperature gradient value agrees with Huitson et al. (2012) using HST

Main result: a set of analytical diagnostics for temperature and number density at line center and line wing

Main takeaway point

(diagnostic to tell if atmosphere is isothermal)

$$\frac{T_1}{T_2} = \frac{\Delta R_1}{\Delta R_2}$$

measure transit radii at line wing
and center for each sodium line
(4 measurements in total)

if ratio is not unity, then atmosphere is non-isothermal!

Summary & Outlook

Heng, Wyttenbach, Lavie, Sing, Ehrenreich & Lovis (2015, ApJ Letters, 803, L9)

- We have derived non-isothermal, analytical diagnostics for extracting temperatures, densities and the temperature gradient from sodium doublet observations of exo-atmospheres.
- To apply them to infrared lines require a better knowledge of pressure broadening.
- Missing physics: photochemistry.

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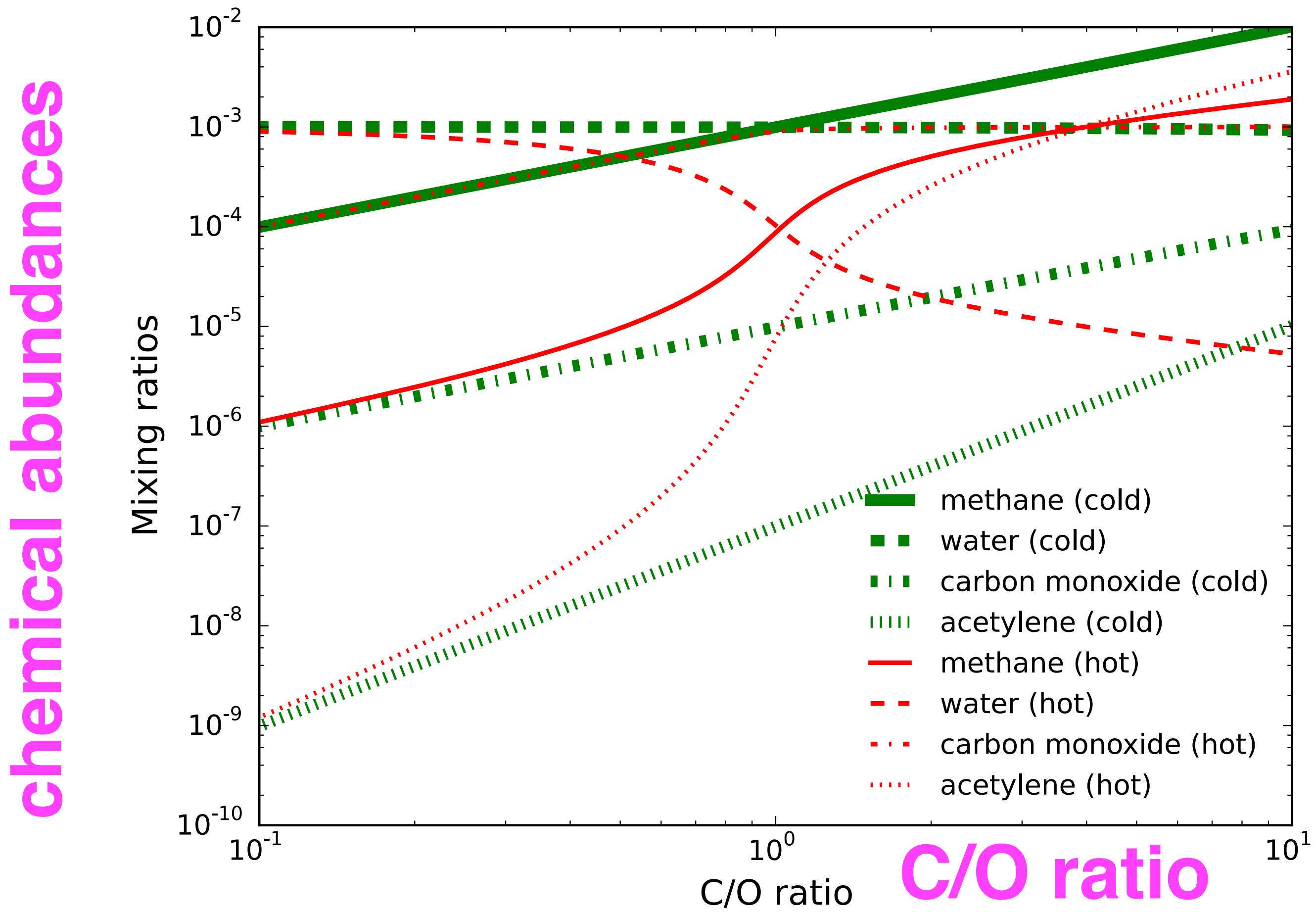
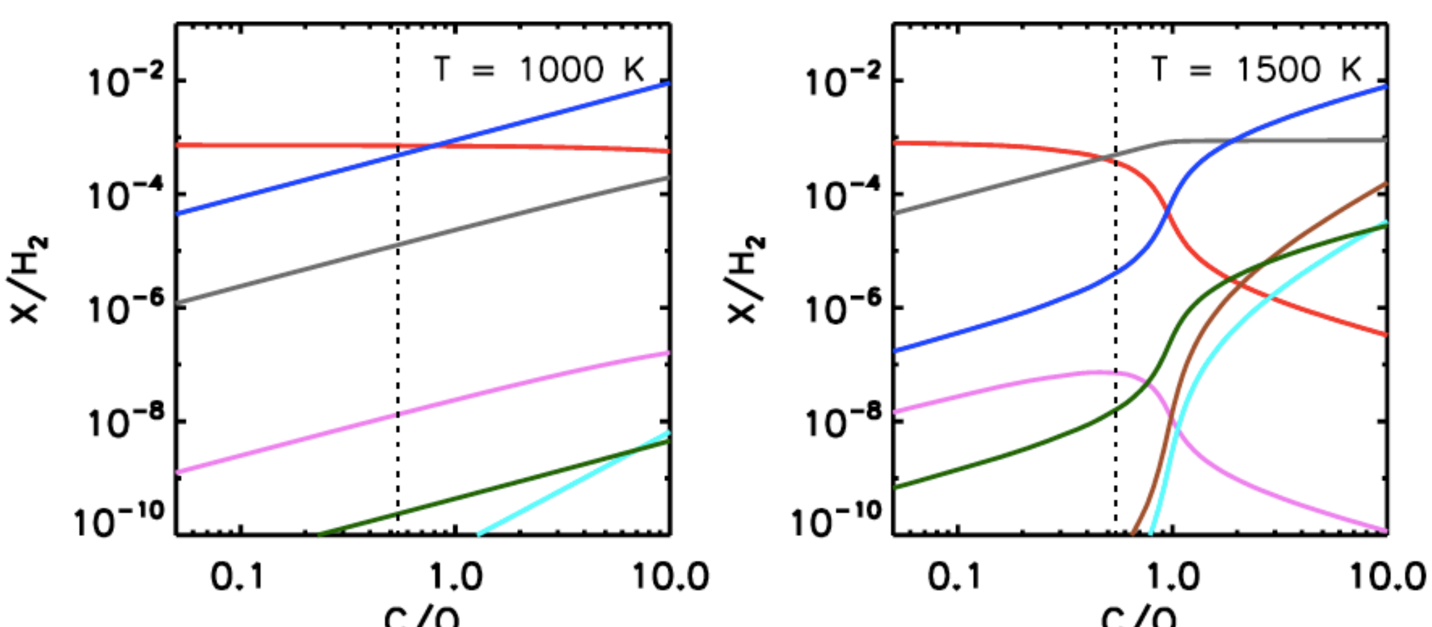
Fresh on arXiv today: new paper on atmospheric chemistry



Heng, Lyons & Tsai (arXiv:1506.05501)



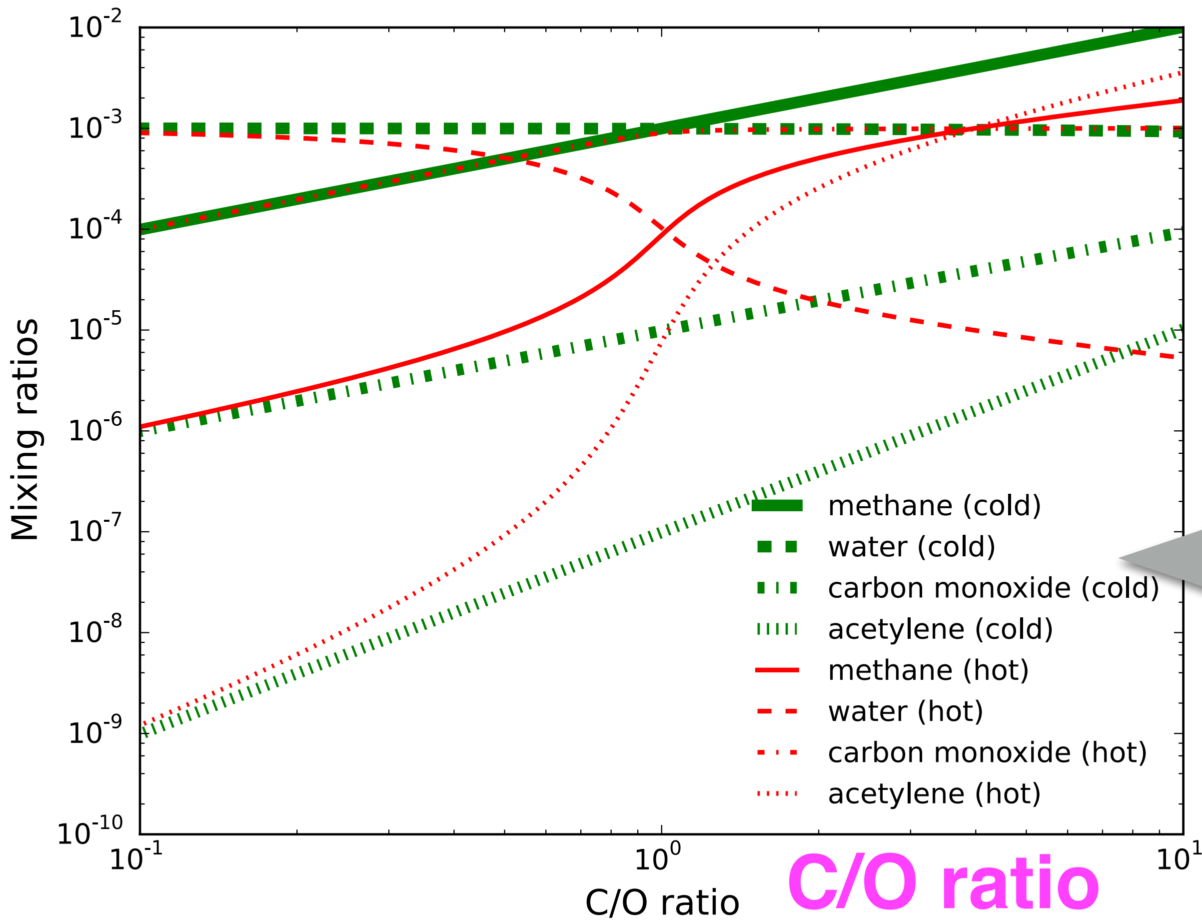
Madhusudhan (2012)

Goal:

to compute chemical abundances as a function of carbon-to-oxygen ratio

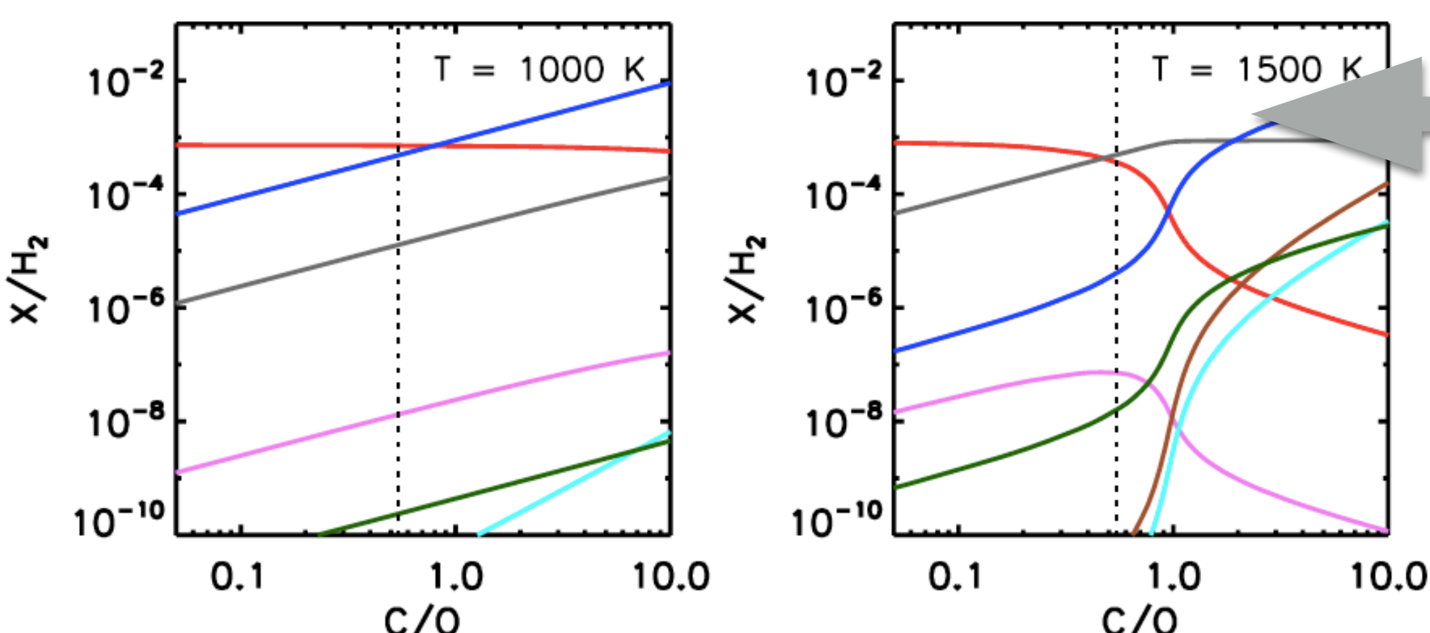
Fresh on arXiv today: new paper on atmospheric chemistry

chemical abundances



C/O ratio

Heng, Lyons & Tsai (arXiv:1506.05501)



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to compute chemical abundances as a function of carbon-to-oxygen ratio

Analytical!

(you don't need a computer)
[Heng, Lyons & Tsai 2015]

Numerical!

(equilibrium chemistry calculation)
[Madhusudhan 2012]

Insight:
rich variety of behaviour is merely stoichiometric book-keeping!