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On some Uncertainties in Evolutionary Synthesis Models

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Abstract. Ranging from track interpolation techniques through model atmospheres to the stochastic nature of the IMF, there are many uncertainties which need to be taken into account when modelling HR diagrams or performing population synthesis, particularly if comparison with actual data is sought. In this paper, we highlight (1) the problem of discontinuities along evolutionary tracks of massive stars (M$>$ 8 $M_\odot$), showing that inconsistencies appear in the computation of the corresponding isochrones, and (2) the sampling fluctuations produced by the stochastic nature of the IMF, presenting a statistical formalism to estimate the dispersion in any given observable of a stellar population due to sampling effects which bypasses the need of performing Monte Carlo simulations.

1. Introduction and motivation

In the last few years, the increasingly detailed observations of stellar populations in a wide variety of environments have provided a huge amount of high quality data, which have been used to constrain both stellar evolution theory and stellar models among many other variables.

In contrast, the development of increasingly complicated evolutionary synthesis codes has, in general, focused only on the use of updated physical ingredients, but some of their underlying hypotheses (track interpolations and isochrone computation in particular) have remained unchanged. In addition, the study of sampling errors has received comparatively little attention (see however Buzzoni (1989), Lançon (these proceedings), and Cerviño et al. (2001b) for a more complete review) even though it seems likely that the scatter in the observed properties of systems with a relatively small number of stars can be accounted for by sampling fluctuations from a given (and perhaps universal) stellar initial mass function (IMF). The aim of this contribution is to highlight some oversimplifications used in these evolutionary synthesis models which lead inevitably to larger uncertainties in some integrated quantities. We present a short general overview of how evolutionary synthesis models work (Sec. §2), the problems raised by discontinuities in stellar tracks of different masses for the isochrones of massive stars (Sec. §3), and the effects of the stochastic nature of the IMF in integrated quantities (Sec. §4).

2. How evolutionary synthesis models work

Irrespective of the actual technique used to obtain the *integrated* properties of a stellar cluster as a function of age several steps must be followed. For illustration we focus here on a fixed metallicity and an instantaneous burst of star formation, and we assume that we have a set of stellar tracks discrete both in the number of stellar masses and in the evolutionary phases provided. As an example, we take here the widely used solar metallicity tracks from Schaller et al. (1992). The *first step* is to interpolate the tracks in order to obtain a set of *continuous* distributions of stellar tracks that describe the same evolutionary sequence for a denser set of initial stellar masses. The *second step* is to assign the age for each mass and each evolutionary sequence in the previously computed stellar tracks. At this stage, the isochrone can be derived from interpolations between the computed tracks at a given age. The next step is to populate the isochrone with the number of stars in a given mass interval. The total number can match the number of stars observed in a cluster, but note that the number of stars in each mass interval is given by the Initial Mass Function (IMF) only *in the asymptotic limit of an infinite number of stars in the cluster.* As an example, we adopt here a power law IMF in the mass range 2–120 $M_{\odot}$ with a Salpeter slope.

Additional complications related to chemical evolution or star formation history introduce other assumptions that will not be dealt with here. It is important to note that although this study is restricted to a narrow range of possibilities (massive stars and young clusters), entirely similar situations are found in older clusters with low mass stars which undergo the Helium flash.

3. Homology relations and isochrone calculations

3.1. Oversimplifications in the use of homology relations

Over the limited region of parameter space where the opacity, κ_0, and the energy production rate, ϵ_0, of the star can be represented by power laws, the internal structure of stars can be assumed to be homologous. This is particularly useful to get, for example, the absolute luminosity, L, as a function of the mass of the star:

$$L(M) \propto \epsilon_0^a \, \kappa_0^b \, \mu^c \, M^d \propto M^d \tag{1}$$

where μ is the mean molecular weight. If the variations of ϵ_0, κ_0 and μ in the tabulated tracks are small enough, interpolations in the $\log L_k - \log M$ plane (where k represents an evolutionary stage) are valid.

Unfortunately, in the case of massive stars it is not always the case. First of all, the initial mass of the star and the mean molecular weight μ are not constant due to the effects of stellar winds. Second, there is obviously a strong discontinuity between the evolution of stars that ultimately become Wolf-Rayet (WR) stars, and which are placed on the left side of the Main Sequence, and stars

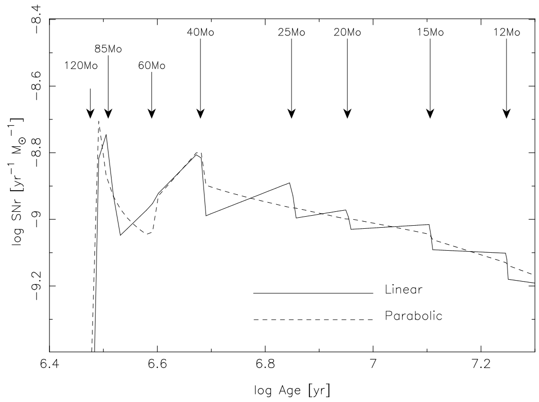

Figure 1. The supernovae rate SNr using two different interpolation techniques. The solid line assumes a linear interpolation in the $\log M - \log t$ plane. The short-dashed line corresponds to a parabolic interpolation.

which become red supergiants. (Note that a similar discontinuity appears for low mass stars which undergo the Helium flash). Such a discontinuity could be solved if the sets of evolutionary tracks would give additional tables corresponding to the appropriate mass range. In the case of Helium flash stars, see for example the solution adopted by Brocato et al. (1999). No such extra information is currently available for this critical range in massive stars, hence there are unavoidable uncertainties associated with this discontinuity.

3.2. Isochrone computations

Assume however that this problem can be bypassed, somehow, and let's go to the next step: a linear interpolation in the $\log M - \log t_k$ plane is used. Such a relation is certainly valid at the Zero Age Main Sequence, but it fails as soon as the stars evolve. This situation is particularly well illustrated by the calculation of the supernovae rate (SNr). If the relation between M and the age where the star becomes a SN, t_{SN}, is a power law, and if the IMF is another power law, the evolution of the SNr must be another power law since it is a convolution of two power laws. We show in Fig. 1 the SNr using linear and parabolic interpolations in the $\log M - \log t_{SN}$ plane.

While the SNr using the linear interpolation exhibits discontinuities corresponding to the discreteness of the stellar tracks, the parabolic interpolation scheme presents a much smoother behaviour. However, independently of the interpolation technique, there are some wiggles at the beginning of the evolution of the SNr due to the particular behaviour of the lifetime of WR stars present in the set of tracks used. However, even if the parabolic interpolation seems to produce better results, a correct interpolation technique (based on physical principles) is still lacking, and a more careful study is required on this subject. A more detailed assessment of this problem can be found in Cerviño et al. (2001a).

4. Stochasticity of the IMF

Besides the problems mentioned in the previous Section, let's turn to the effect of sampling the IMF to evaluate the dispersion due to the discreteness of actual stellar populations. Let us assume that N_{tot} stars are observed, with masses distributed between M_{low} and M_{up}. The probability distribution function of the stellar masses is given by the IMF, so that the number of stars N_i of a given mass M_i is a random variable with a Poissonian distribution (see Cerviño et al. 2001b for details).

As pointed out by the pioneering study of Buzzoni (1989), the fact that N_i is a Poisson variable makes it possible to apply a proper statistical formalism. Let us consider for instance the integrated monochromatic luminosity L_λ of a stellar population of N_{tot} stars of a given age t. The average value of L_λ is given by the sum of the individual monochromatic luminosities $l_{\lambda,i}$ of stars of age t belonging to the i^{th} mass, each weighted by its number as given by the IMF, $w_i = N_i/N_{tot}$:

$$< L_\lambda > = N_{tot} \sum_i w_i(t)\, l_{\lambda,i}(t). \tag{2}$$

Up to a constant, $L_\lambda(t)$ will be Poisson distributed, since the sum of Poisson variables is also a Poisson variable, with parameter given by the sum of the individual Poisson parameters. Due to random fluctuations in the number of stars of each mass, the actual luminosity L_λ will fluctuate around this average value with a variance given by

$$\sigma^2(L_\lambda) = < (L_\lambda - < L_\lambda >)^2 > = N_{tot} \sum_i w_i\, l^2_{\lambda,i} \tag{3}$$

and thus the dispersion due to the discreteness of the stellar population is obtained for any quantity that is derived from the integrated luminosity, such as colours or the number of ionising photons. Note however that for *derived* quantities one has to take into account the *covariances* between the w_i, weighted by their luminosities. This can be illustrated by the correlation coefficient of two quantities $\rho(x, y)$ which is defined as:

$$\rho(x, y) = \frac{cov(x, y)}{\sigma_x\, \sigma_y}. \tag{4}$$

It obviously varies between -1 and $+1$, where the sign indicates the sense of the correlation. So $\|\rho(x, y)\| = 1$ if the quantities are completely correlated and $\rho(x, y) = 0$ if there is no correlation. For a given star, the luminosities in different bands are completely correlated since they are produced by the same star. But the situation changes when several stars are considered simultaneously, such as in a cluster. At some ages, the same stars will contribute to the same bands, and hence the correlation coefficient will be unity for these bands, but at some later stages different masses will contribute differently and the correlation will decrease. As an illustration, Fig. 2 show the evolution of the correlation coefficient for some optical and NIR bands. Neglecting this evolving coefficient may produce an underestimation of the sampling errors. Similarly some bands

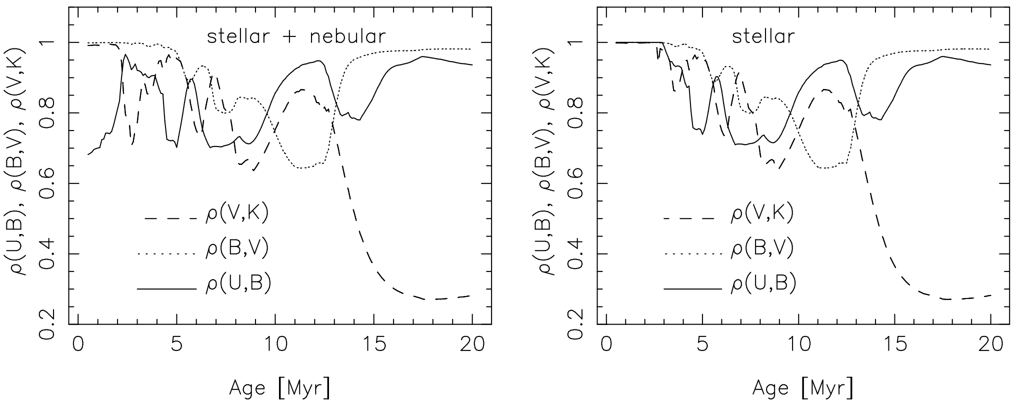

Figure 2. *Correlation coefficients.* Evolution of the correlation coefficients for U and B, B and V and V and K for a cluster where the nebular contribution is included (left panel) and for a cluster with only stellar contribution (right panel).

may be more affected by the nebular continuum (strongly correlated to the more massive stars) than others, and hence the actual dispersion on a given magnitude or colour will depend on the precise correlation coefficient, as shown on Fig. 2.

Using simple rules of error propagation, and taking into account the covariance terms, we can derive *analytically* the dispersion in any integrated quantity derived from the luminosity. Fig. 3 shows the interesting case of the $V-K$ colour as an illustration. We have performed Monte Carlo simulations with 1000 clusters with N_{tot}=10^3 stars, 500 clusters with 10^4 stars and 100 clusters with 10^5 stars. In each set we have obtained the dispersion σ_{clus}(V–K). The σ_{clus} values have been divided by $N_{tot}^{0.5}$, to obtain a normalized effective dispersion for each set (see Cerviño et al. 2001b for further details).

The top left panel shows the mean $V-K$ colour index for different simulations and the analytical one (the idealised case of an infinite number of stars). The comparison shows that our analytical estimation performs very well, although there are slight deviations in the 10^3 stars cluster simulations. The low left panel shows the comparison of the normalized values of $\sigma_{clus}(V-K)$ with the analytical value. It shows that the analytical computation of the dispersion and the Monte Carlo results are again very similar except for the 10^3 star clusters. The differences are due to an additional dispersion introduced by the relation between N_{tot} and the total mass of the cluster. We also show the 90% confidence levels of the different simulations computed from the Monte Carlo simulations (shadowed areas) and analytically (lines). It is important to stress that these confidence level bands are not *symmetric* with respect to the mean value, as expected for quantities derived from a Poisson distribution. We refer the reader to Cerviño et al. (2001b) for more details.

5. Conclusions

Evolutionary synthesis models clearly have oversimplifications that must be studied in much more detail if reliable predictions are to be made. Some of

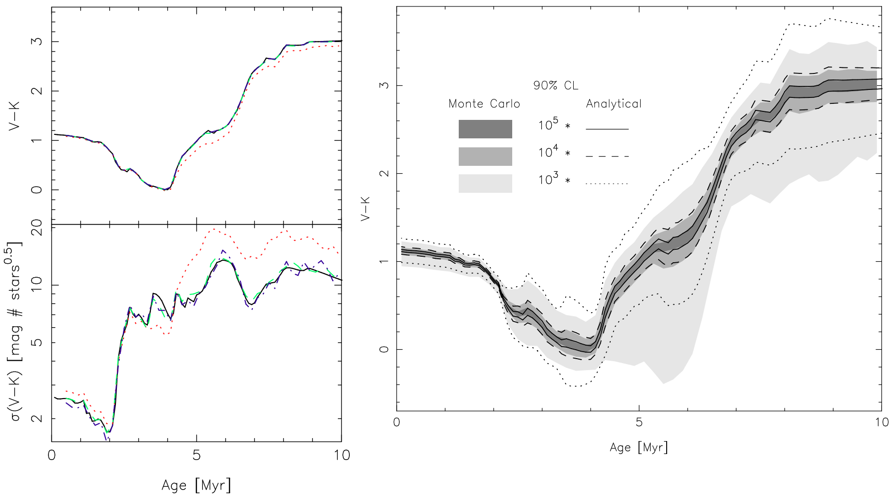

Figure 3. *Comparison of analytical versus Monte Carlo dispersion estimations. Top left:* Evolution of the average $V-K$ color index for the analytical (solid line) and Monte Carlo simulations (dashed/dotted). *Bottom left:* Comparison of normalised dispersions $\sigma(V-K)$ for analytical and Monte Carlo estimations.*Right:* Comparison of the analytical estimation of the confidence levels (lines) with Monte Carlo ones (shadowed areas).

them can be solved, at least in part, if evolutionary tracks covering a denser grid in mass are published. The effects of the discreteness of stellar populations in small systems must also be considered before a comparison of model predictions to real data is attempted.

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